Policy Approval Date: September 2024

Review Date: September 2025

This policy was written with guidance from Cambs Maths Advisors with the Maths Co-ordinator in April 2024. This policy is reviewed each year by the Maths Subject Leader.

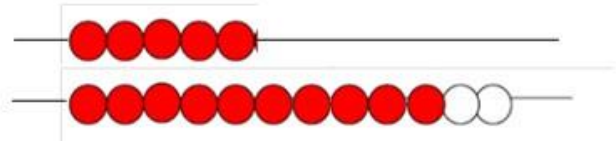
This policy outlines the progression through **written strategies** for addition, subtraction, multiplication and division, which work alongside visual resources to represent the final written strategy. The aim is for children to become fluent in these written strategies through varied and frequent practice so that pupils develop conceptual understanding and the ability to recall and apply knowledge in a range of contexts. In addition to **fluency** of calculation teachers also need to consider frequent opportunities for children to **reason** using mathematical language as well as **solve problems** by applying their mathematics.

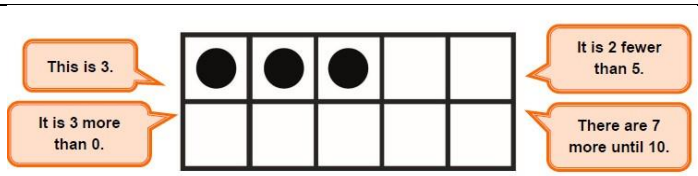
Children will move through the stages of written calculation at the pace appropriate to them however we expect the majority of each class to be working at age-appropriate levels as set out in the National Curriculum 2014. The policy includes examples and diagrams showing how to teach calculations as consistency in layout and presentation is important. The policy also includes the equipment and resources that will be used to support children's understanding of each strategy.

This policy focuses on written calculation in maths. It is also important to teach mental strategies alongside, which is done in accordance with the White Rose documents and Flash Back Four fluency learning.

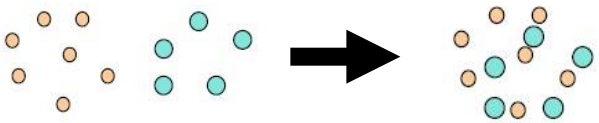
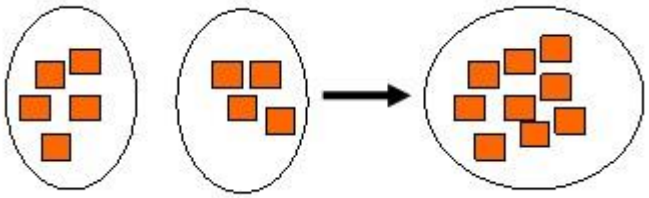
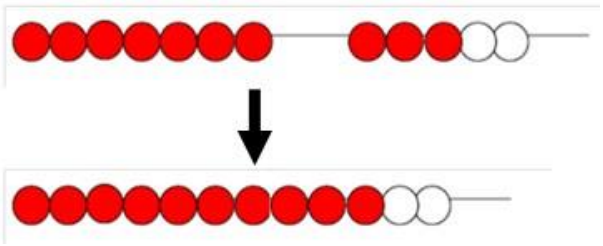
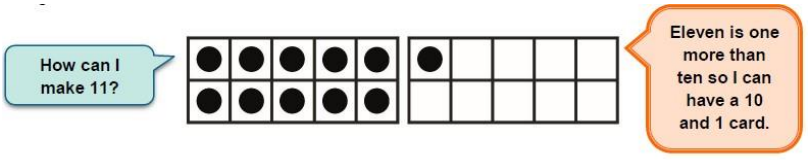
Counting and partitioning

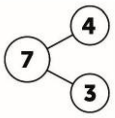
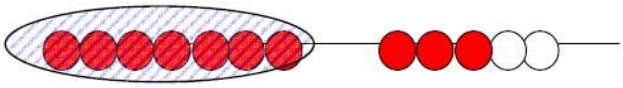
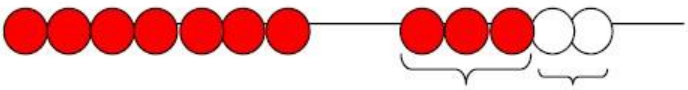
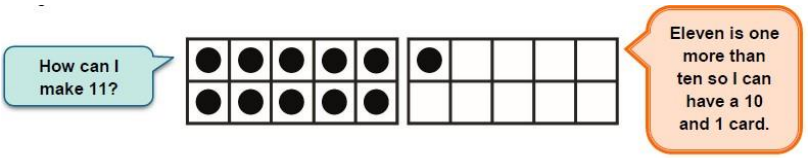
Counting: 1. One to one correspondence	1. Children synchronise their counting and pointing, keeping track of their counting as they go, assigning one number name to one object and only counting each object once. Counting static pictures is harder and children need to devise a system to know which they have counted as they go along.
2. Stable order of counting	2. To be able to count means knowing that the list of words used must be in a repeatable order. This principle calls for the use of a stable list that is at least as long as the number of items to be counted; if children only know the number names up to 'six', then they obviously are not able to count seven items.
3. Cardinal aspect of number	3. This is the idea that when they are counting a set of objects, the last number counted is the number of objects altogether.
4. Abstract principle of number	4. This is where children are counting things that cannot be touched or moved, such as sounds, imaginary objects or even the counting words.

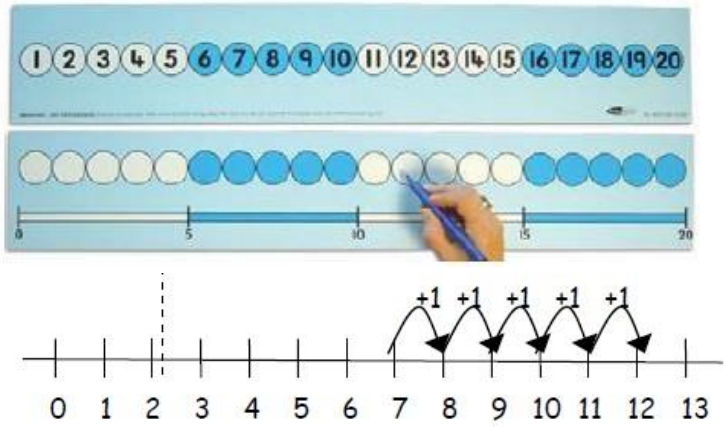
Greater than/less than/equivalent	Using direct comparison with manipulatives – which is more? Bead strings:  Numicon:  Balance scales: 
Partitioning	1. Complements to 1, 10, 100 Bead string:  Numicon:  2. Partitioning any number in all possibilities e.g. partitioning 9: 0 + 9 1 + 8 2 + 7 3 + 6 4 + 5

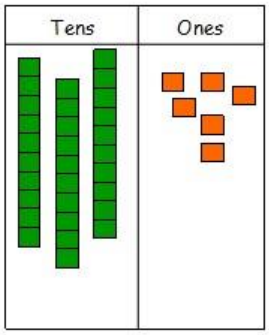
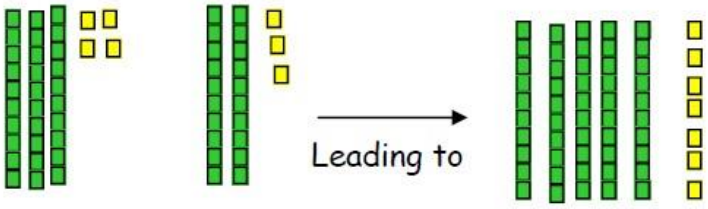
	 3. Tens frame, partitioning to ten and number bonds to ten.

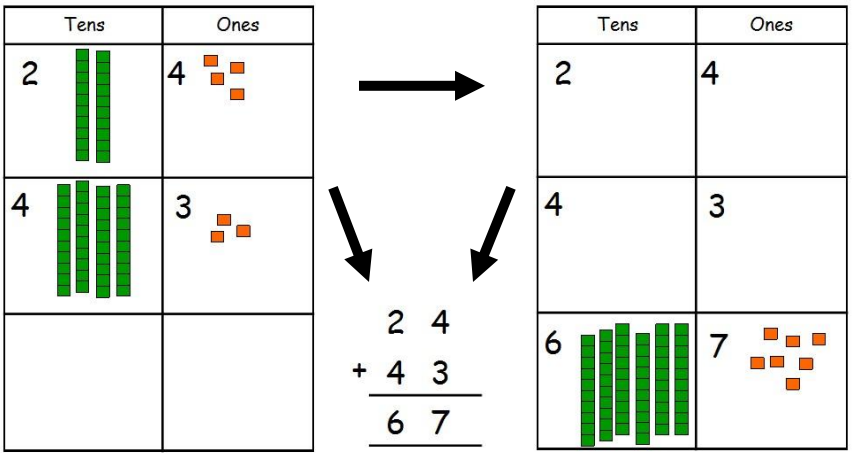
Addition

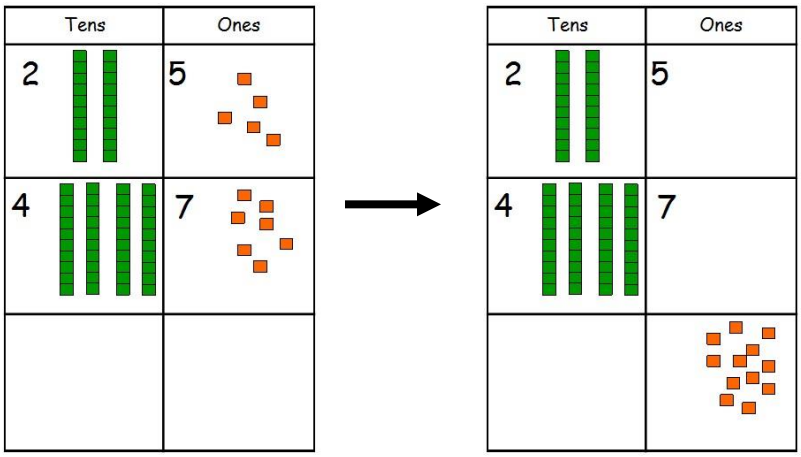
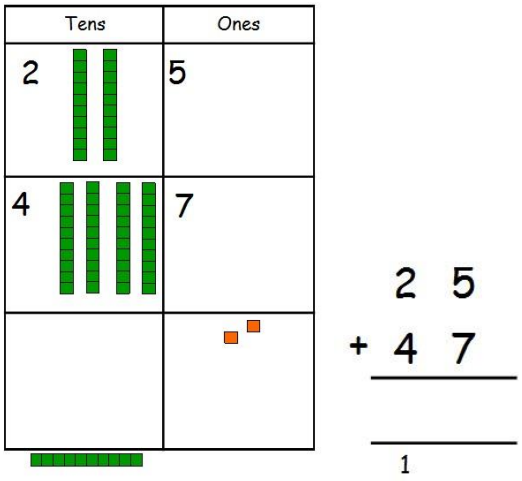
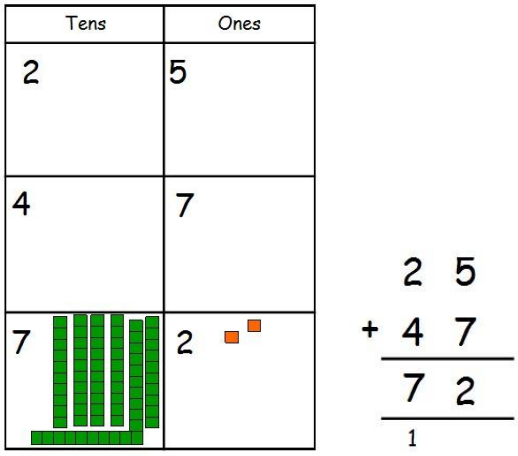
Combining and counting (aggregation)	Count one set, then the other. Combine the sets and count again starting from 1. Using mixed sets of objects:  Count each set Combine and count Dienes units:  Count each set Combine and count Bead strings:  Count each set, combine and count Tens Frame 

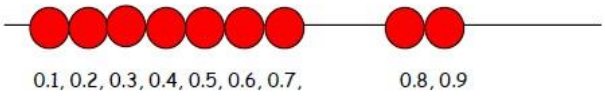
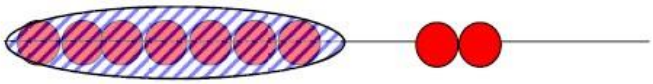
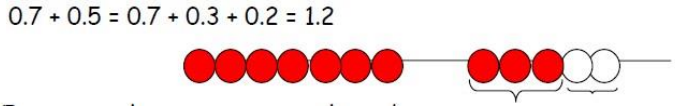
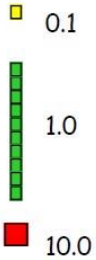
	  Part/Whole cherry model
Combining and counting *on* (augmentation) **NB. IMPORTANT NOT TO FORGET THIS STEP**	Count one set, then count on from that set. Prepare for this stage with lots of play – count one set, hide it in bags, behind back, teddy eating them etc. Before counting on. Bead string:  Count 7, then count on 8, 9, 10, 11, 12  Starting on ten, and then counting up
Bridging through 10	1. Manipulative – bead string:  7 + 5 How many more to the next multiple of 10? 3 If we use 3 of the 5 to get to 10 how many more do we need to add on? 2. Tens Frame   Link to

	number line 
Transition to Dienes	Familiarise the children with Dienes: 1. Compare 10s and 1s – lay units along the track of a ten. They need to grasp the relationship and equivalence.

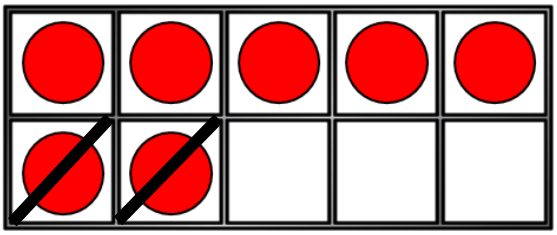
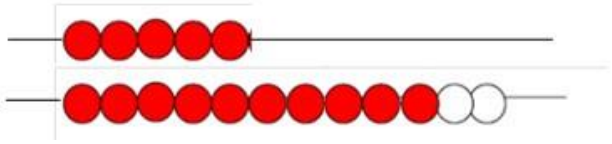
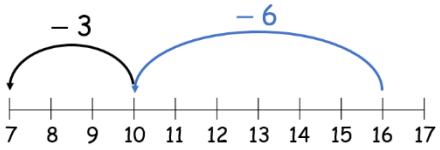
	2. Partition Tens and Ones using place value charts  3. Aggregation – combine the two sets, count <u>ONES FIRST</u>  (starting from one), then tens (starting from ten) 4. Augmentation – as above but count from the first set of ones and tens, avoid starting at 1 i.e. start at 4 then continue 5, 6, 7.

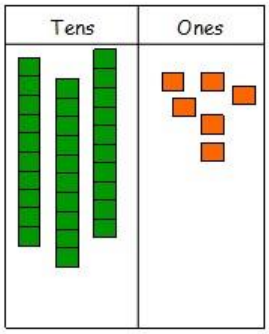
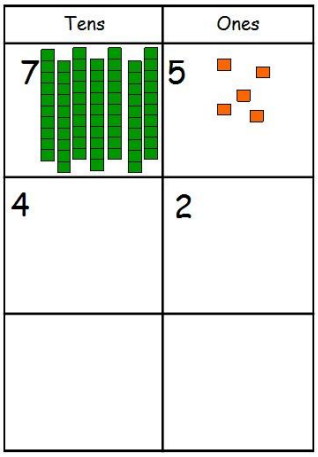
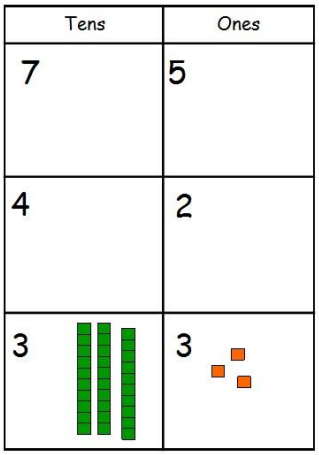
Columnar recording – no exchange	Recorded on place value charts with Dienes, leading to without Dienes.  Combine **<u>Ones first</u>**, count and record, then Tens. **NB. For children who are struggling to understand this method, teachers may decide to introduce the expanded method**

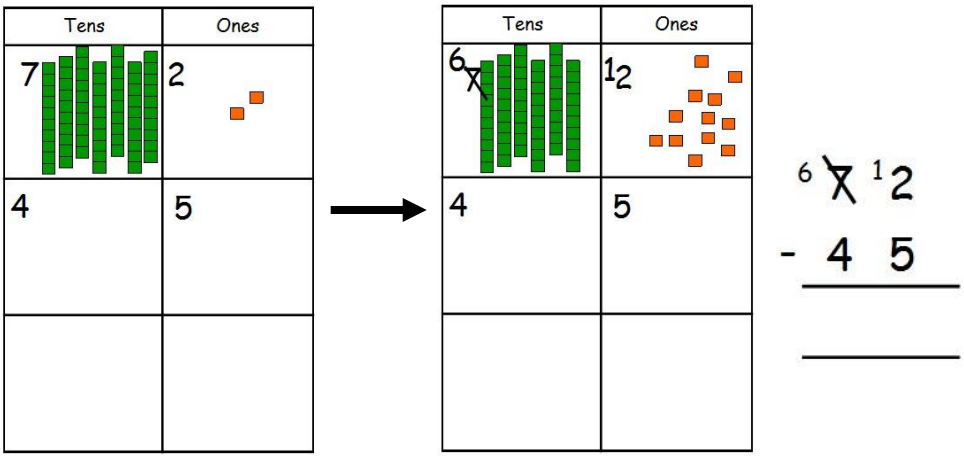
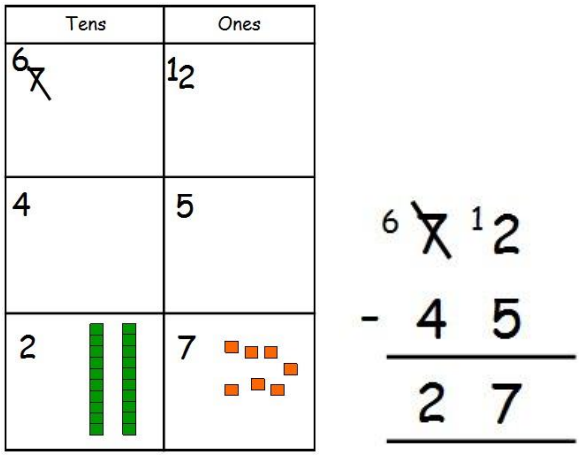
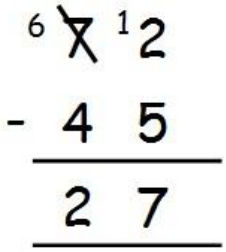
Columnar recording – exchanging	 Combine Ones  Exchange ten Ones for one Ten Move the Ten to the next column – carrying underneath  Combine tens Run Dienes and place value chart method alongside compacted written method until children are confident.

Compacted written method	2 5 + 4 7 7 2 1 When children are confident without Dienes. Grading of difficulty: 1. No exchange 2. Extra digit in the answer 3. Exchanging Ones to Tens 4. Exchanging Tens to Hundreds 5. Exchanging Ones to Tens AND Tens to Hundreds 6. More than two numbers in calculation 7. Different numbers of digits 8. Decimals – see next stage
Decimals	It is important to take children back through the stages with decimals: Aggregation: Count both sets starting from zero  Augmentation: art from 0.7, count on 0.8, 0.9  Bridging through 1:  Columnar method with Dienes as decimals. Then, written column method as above 

Subtraction

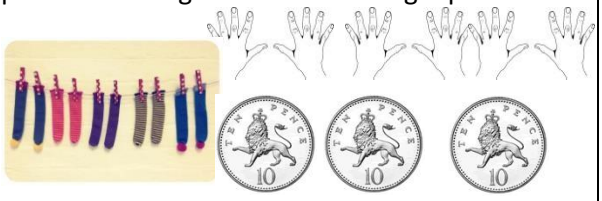
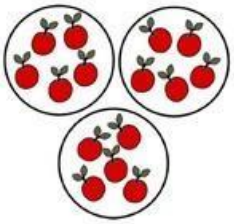
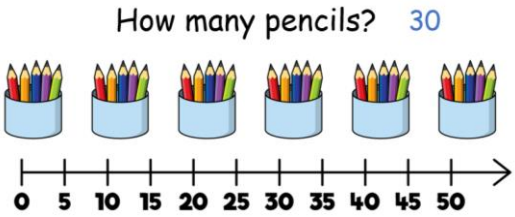
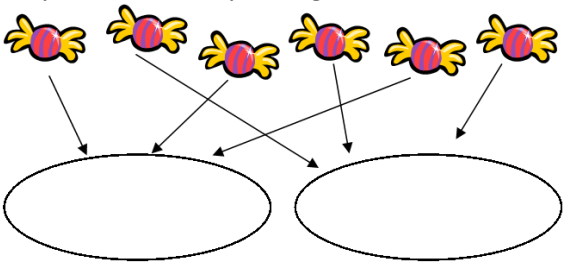
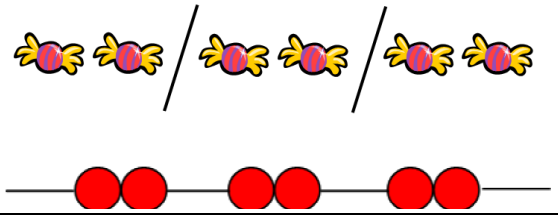
3 models of subtraction to be used throughout	**1. <u>Take away/count back</u>**  Seven subtract 2 7 objects, count back 2 **<u>Comparing two sets/difference</u>**  a. – 5 Comparing 12 and 5, count difference
Subtracting single digits	• Use manipulatives in the above 3 ways (numicon, tens frame) • Visualise manipulative in head • Pictorial representation of one of the manipulatives • Link to number line 
Transition to Dienes	Familiarise the children with Dienes: 1. Compare 10s and 1s – lay units along the track of a ten. They need to grasp the relationship and equivalence.

	2. Partition Tens and Ones using place value charts 
TO – TO using Dienes and columnar place value charts – no exchange	 7 5 - 4 2 Take the Ones first, then Tens  7 5 - 4 2 3 3 Record the answer underneath after moving the Dienes **NB. For children who are struggling to understand this method, teachers may decide to introduce the expanded method**

TO – TO using Dienes and columnar place value charts – with exchange	 Exchange one Ten for ten Ones, alter written numbers  Take the **Ones**, *then* take the **Tens** **Once children are confident with the practical – record the compacted method *alongside* as shown.**
Column method	When children are confident with Dienes and compacted method alongside, begin to record without Dienes.  Grading of difficulty: 1. TO – TO, no exchange 2. TO – TO with exchange 3. HTO – TO with exchange 4. HTO – HTO with exchange 5. HTO – HTO with a zero in Tens column 6. Larger numbers

	7. Decimals: Same as above but reattributing values to dienes

Multiplication and Division

Multiplication	**Division**
Early experiences Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets  or groups.	Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s. 
Repeated addition (repeated aggregation) 3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3 Children learn that repeated addition can be shown on bead string: 	**Sharing equally** 6 sweets get shared between 2 people. How many sweets do they each get? 
	Grouping of repeated subtraction (NB important step – often forgotten) There are 6 sweets. How many people can have 2 sweets each? 

Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you end up with 15 marbles. This can be written as:

1 + 1 + 1 = 3

5 + 5 + 5 = 15

5 x 3 = 15

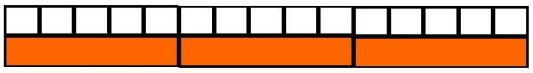
Repeated subtraction

Using a bead string, number track, then number line.

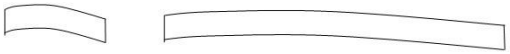
12 ÷ 3 = 4

How many 3s make 12?

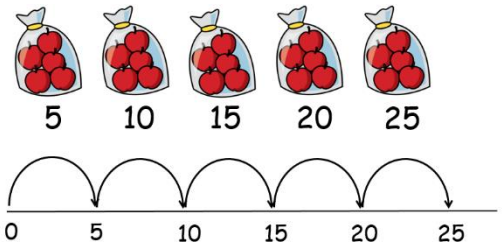
1 + 1 + 1 = 3 ➐ Scaled up ➐ 5 + 5 + 5 = 15



For example, find a ribbon that is 4 times as long as the blue ribbon.



5cm ribbon 20cm ribbon

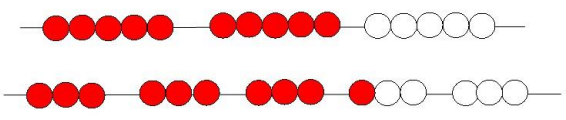


It is also important for children to have experience of different ways of this partitioning e.g. How many different ways can we make 12?

Commutativity

Children learn that 3 x 5 has the same total as 5 x 3. This can be shown on bead strings and number lines.

3 x 5 = 15



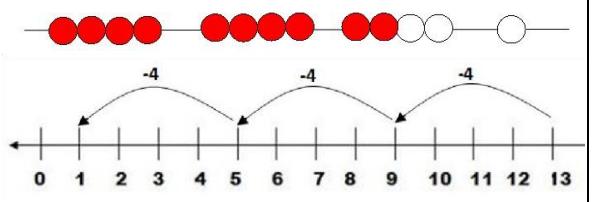
5 x 3 = 15

Grouping involving remainders

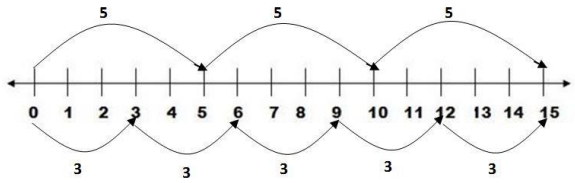
Children move onto calculations involving remainders.

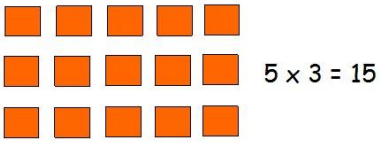
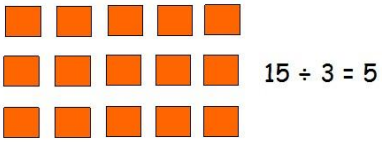
If there are 13 sweets shared between 4 children, how many would each child get?

13 ÷ 4 = 3 r 1



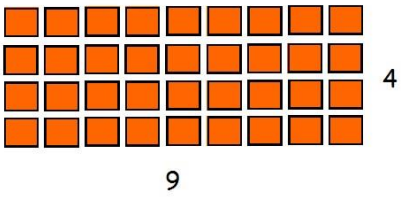
Remainders should be given as integers, but children need to be able to decide what to do

	after division, such as rounding up or down accordingly. E.g. I have 62p. How many 8p sweets can I buy? E.g. Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?
Arrays Children learn to model a multiplication calculation using an array. This model supports their understanding of commutativity and the development of the grid in a written method.	**Arrays** Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method.

 3 x 5 = 15 **Children should be taught to arrange the array in different ways e.g. How many different ways can they be arranged?**	 15 ÷ 5 = 3 **Children should be taught to arrange the array in different ways e.g. How many different ways can they be arranged?**

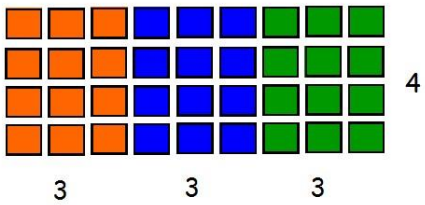
Partitioning and the distributive law

Arrays are useful to help children visualise how to partition larger numbers into more useful arrays.



9 x 4 = 36

Children could break this down into more manageable arrays (distributive law – New curriculum Y4):

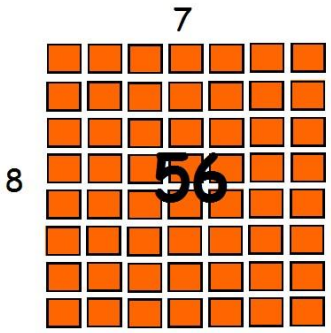


9 x 4 = (3 x 4) + (3 x 4) + (3 x 4)
= 12 + 12 + 12
= 36

Children need to spend lots of time investigating different ways to partition using low numbers to embed this concept. (Children can use multilink cubes, Dienes blocks, squared paper to make and split the arrays.)

Partitioning and the distributive law

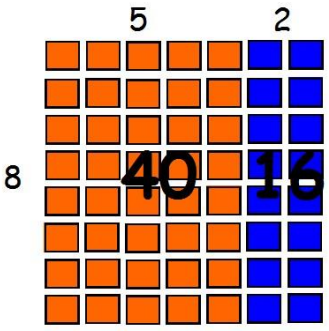
Arrays are useful to help children visualise how to partition larger numbers into more useful arrays.



56 ÷ 8 = 7

Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between multiplication and division. (Distributive law – New curriculum Y4):

56 ÷ 8 =

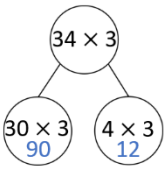


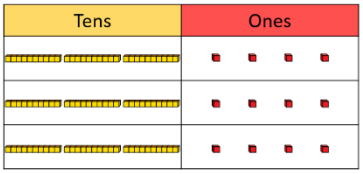
56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8)
= 5 + 2
= 7

Children need to spend lots of time investigating different ways to partition using low numbers to embed this concept. (Children can use multilink cubes, Dienes blocks, squared paper, to make and split the arrays.)

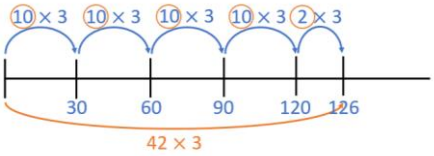
Use of Dienes and number lines to represent part/whole as part of multiplication

34 × 3 = 102



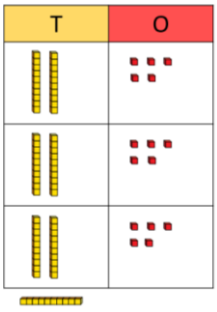


42 × 3 = 126



Expanded short method, including regrouping:

3 × 25 = 75

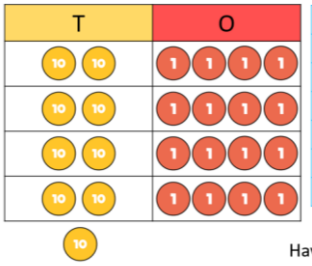


3 × 5 = 15

3 × 20 = 60

15 + 60 = 75

4 × 24 = 96

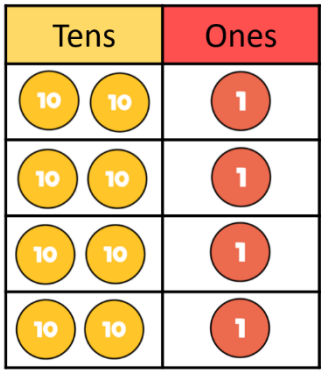


	T	O	
	2	4	
×		4	
	1	6	(4 × 4)
	8	0	(4 × 20)
	9	6	

Have a think

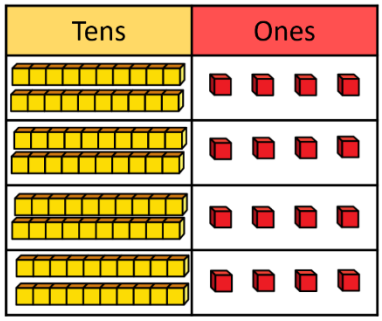
Use of Dienes/Counters for grouping towards the short method:

84 divided by 4



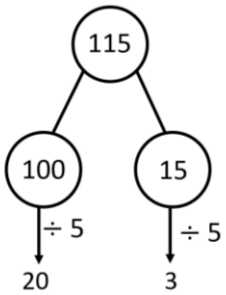
With partitioning within part/whole model:

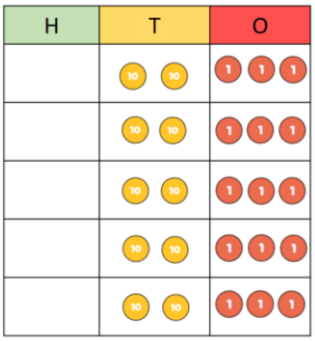
96 ÷ 4 = 24



Use of the Part/Whole model to aid division:

115 ÷ 5 = 23

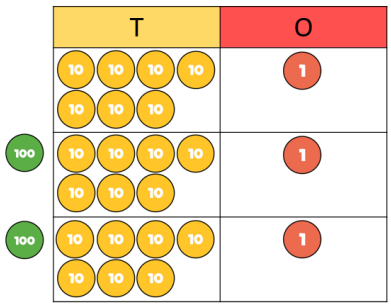




Short method for Multiplication

$3 \times 71 = 213$

Have a think



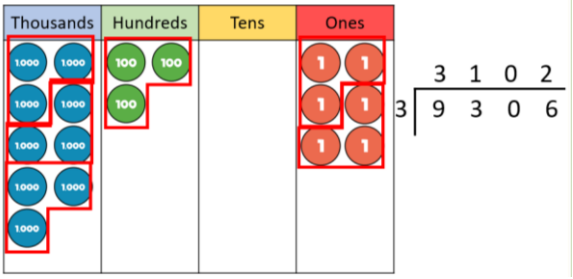
	T	O	
	7	1	
×		3	
2	1	3	
2			

$$\begin{array}{r} 24 \\ \times \quad 6 \\ \hline 144 \\ \hline 2 \end{array}$$

Short method for division

9,306 ÷ 3

Have a think



$$\begin{array}{r} 26 \\ 3\overline{)\,7^{1}8} \end{array}$$

432 ÷ 5 becomes

$$\begin{array}{r} 86\ r2 \\ 5\overline{)\,432} \\ \quad 3 \end{array}$$

Grading of difficulty (short multiplication)

1. TO x O no exchange
2. TO x O extra digit in the answer
3. TO x O with exchange of ones into tens
4. HTO x O no exchange
5. HTO x O with exchange of ones into tens
6. HTO x O with exchange of tens into hundreds
7. HTO x O with exchange of ones into tens and tens into hundreds
8. As above with greater digits x O
9. O.t x O no exchange
10. O.t with exchange of tenths to ones
11. As above with greater numbers of digits and decimals places.

Grading of difficulty (short division)

1. TO ÷ O no exchange no remainder
2. TO ÷ O no exchange with remainder
3. TO ÷ O with exchange no remainder
4. TO ÷ O with exchange, with remainder
5. Zeros in the quotient e.g. 816 ÷ 4 = 204
6. As 1 – 5 HTO ÷ O
7. As 1 – 5 greater number of digits ÷ O
8. As 1 – 5 with a decimal dividend e.g. 7.5 ÷ 5 or 0.12 ÷ 3

Grading of difficulty for expressing remainders

1. Whole number remainder
2. Remainder expressed as a fraction of the divisor
3. Remainder expressed as a simplified fraction
4. Remainder expressed as a decimal

Long multiplication

Children will refer back to the grid method and compare before recording as:

Long division

432 ÷ 15 becomes

```
      2  8  r 12
   ┌──────────
1 5 │ 4  3  2
    - 3  0  0
    ──────────
      1  3  2
    - 1  2  0
    ──────────
         1  2
```

```
  1074
x   22
  ----
     8
   140
   000
  2000
    80
  1400
  0000
 20000
 -----
 23628
```

```
   1 1
  1074
x   22
 ----
 2148
21480
-----
23628
  1
```

Children may then be taught that long multiplication can be done in a different order, depending on whether you start with ones or tens/hundreds. Children are encouraged to understand that both methods have the same processes.

24 × 16 becomes

```
   2
  24
× 16
----
 240
 144
----
 384
```

Answer: 384

124 × 26 becomes

```
   1 2
  124
×  26
-----
 2480
 744
-----
3224
 1 1
```

Answer: 3224